Stable Torsion Length

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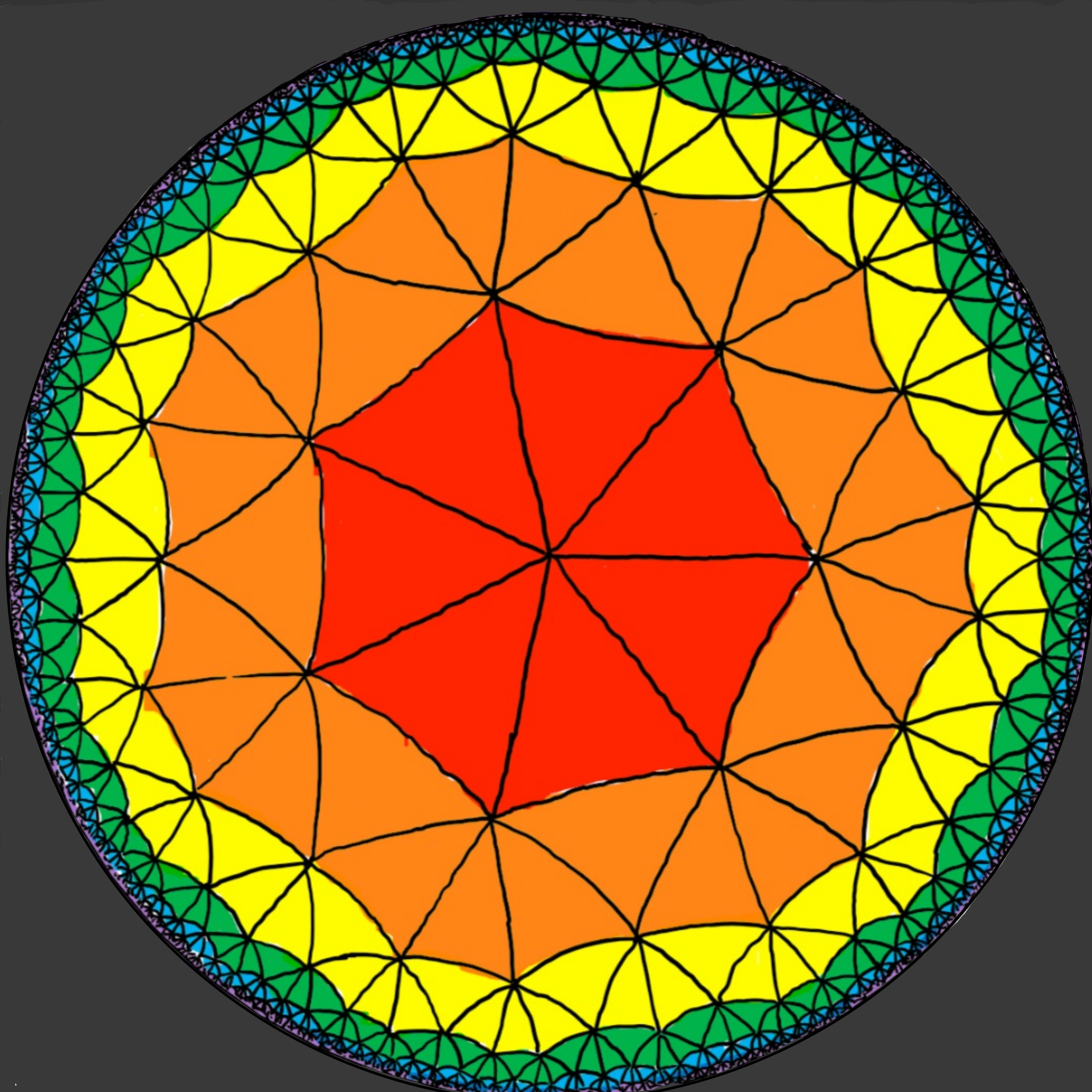
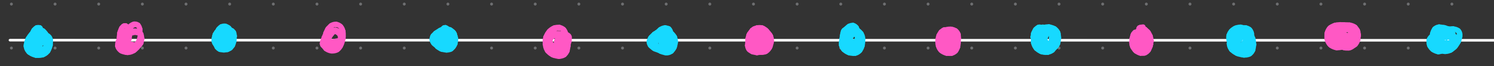


$\boxed{\text{Defn}}$ G group, $g \in G$, the <u>torsion length</u> of g is:

$$tl(g) := \min\{n \mid g = t_1 \cdots t_n,\ t_i \text{ torsion}\}$$

Is torsion length bounded in G?
[Brendle - Farb 2003]

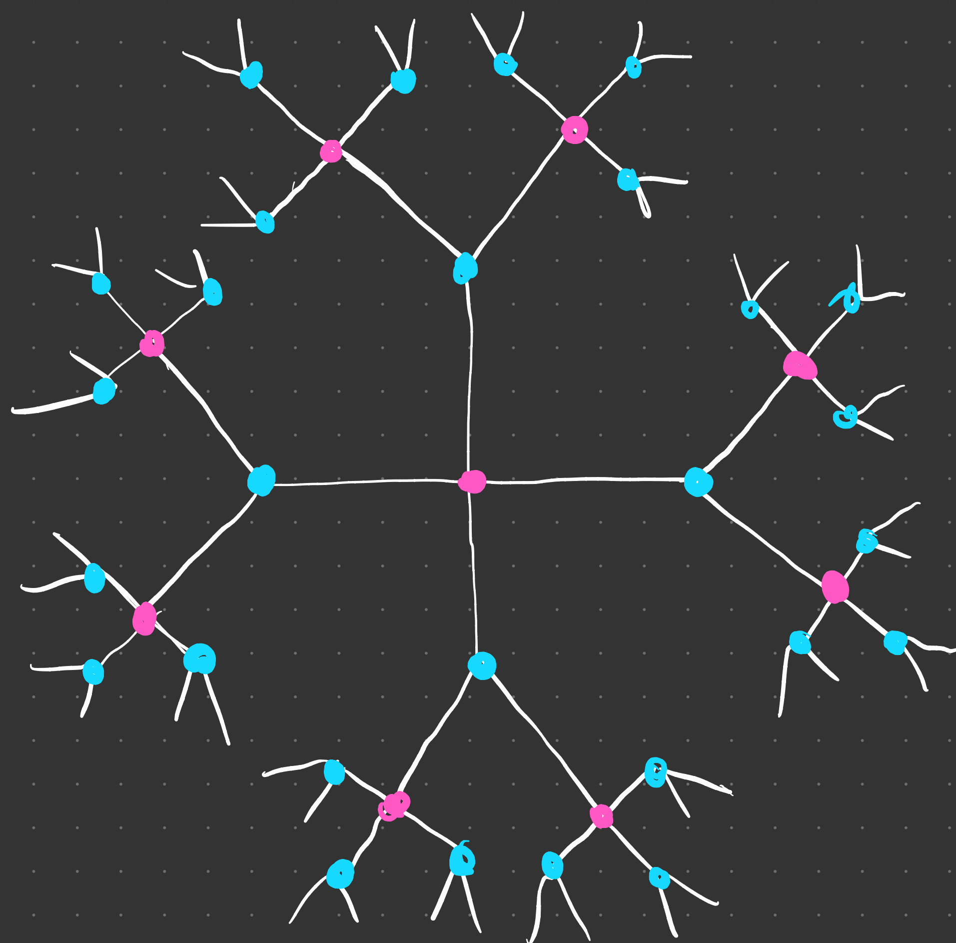
<u>Example:</u> $G = \langle a, b \mid a^2 = b^2 = 1 \rangle = \mathbb{Z}/2 * \mathbb{Z}/2$



$$(ab)^n = \underbrace{(ab\dots ba)\, b\, (ab\dots ba)}\ \underbrace{b}$$

$$tl(g) \leq 2 \quad \forall g \in G$$

<u>Example:</u> $G = \langle a, b \mid a^3 = b^4 = 1 \rangle = \mathbb{Z}/3 * \mathbb{Z}/4$



$$abab = \underbrace{aba^{p-1}}\,\underbrace{a^2}\,\underbrace{b}$$

Torsion length in G is unbounded.

$$tl((ab)^2) \leq 3$$

How to know if this is the best I can do?

When torsion length is unbounded:

$\boxed{\text{Def}^n}$ G gen by torsion. The <u>stable torsion length</u> of $g \in G$ is

$$\mathrm{stl}(g) := \lim_{n\to\infty} \frac{\mathrm{tl}(g^n)}{n}$$

$\boxed{\text{Thrm}}$ [Kotschick, 2003]

G gen by torsion, $g \in G$, then

$$\mathrm{stl}(g) \geq 2\,\mathrm{scl}(g)$$

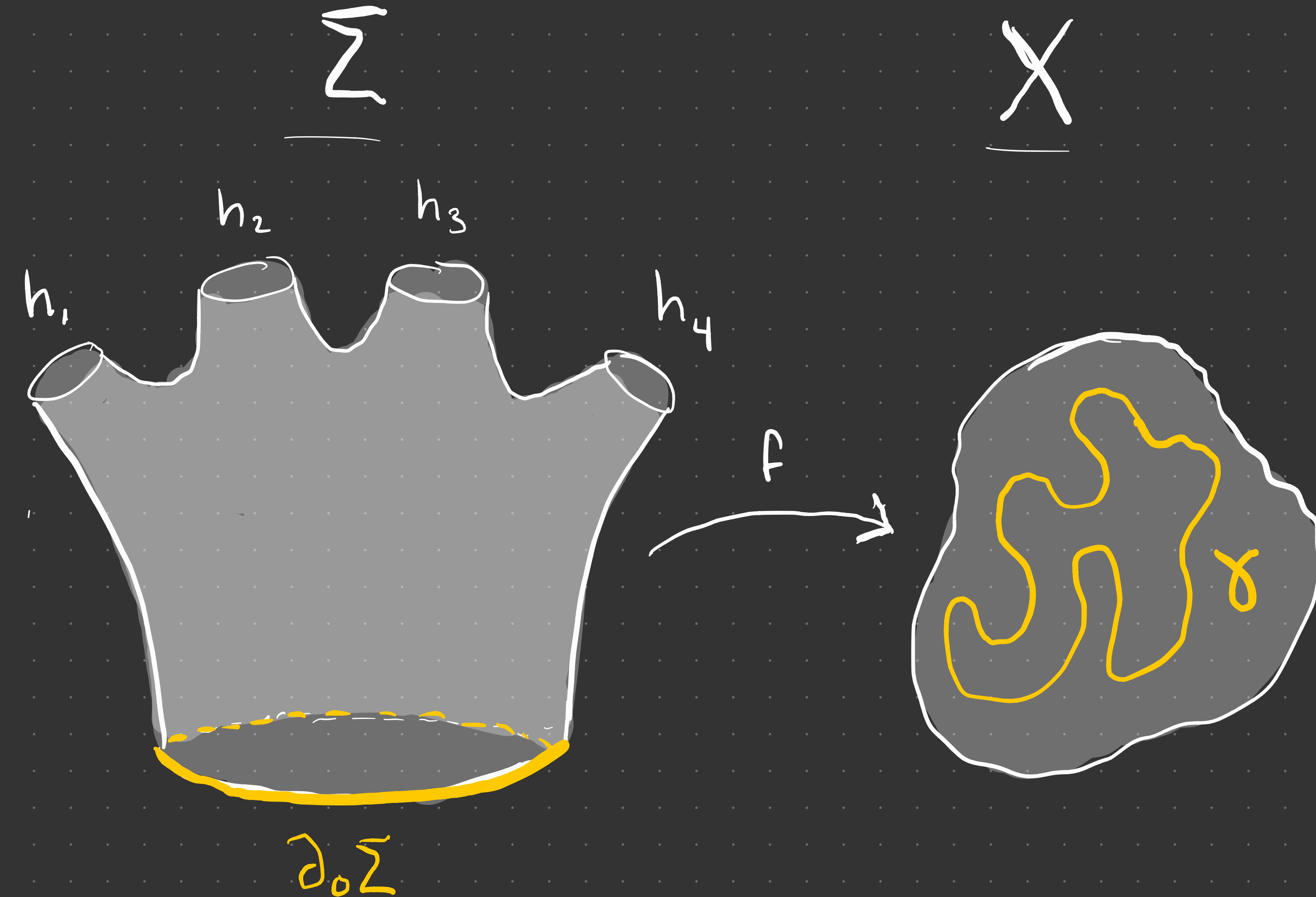
proof: uses quasimorphisms.

In general, stl is hard to compute.

$\to$ Use topology!

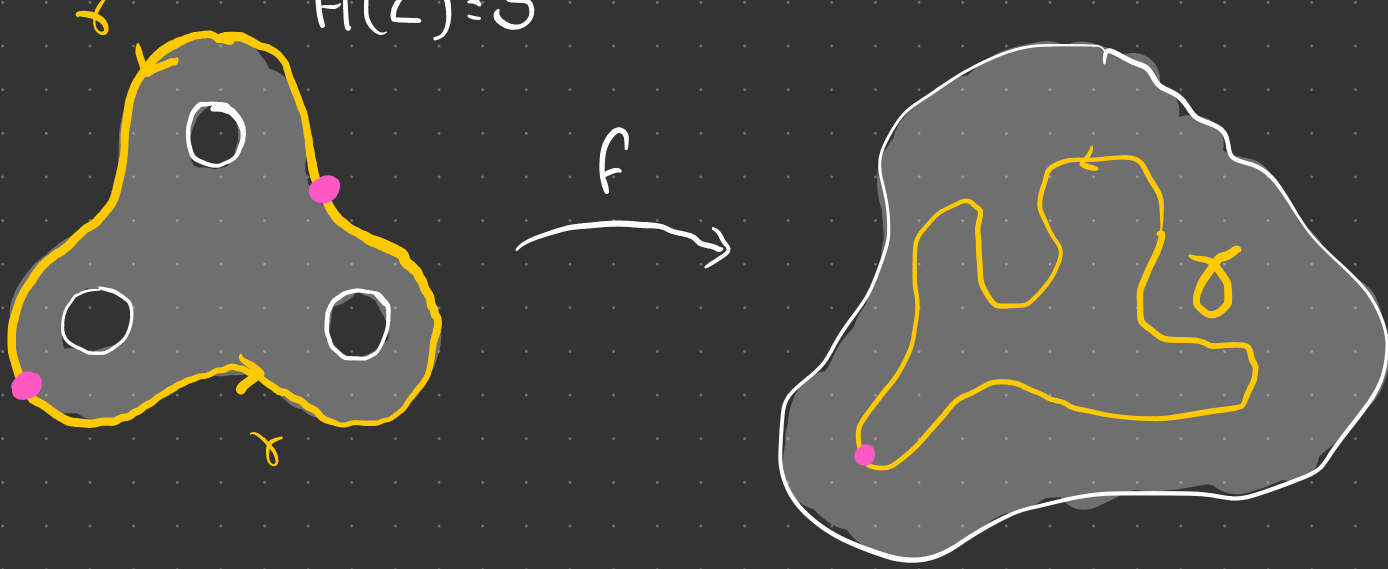
$X :=$ CW complex $\quad \pi_1(X) = G$

$[\gamma] \in \pi_1(X)$ want: $\mathrm{stl}([\gamma])$



Consider maps $f : \Sigma \to X$

$H(\Sigma) = 3$



$\deg(f|_{\partial_0 \Sigma}) = 2 \quad H(\Sigma) = 3$

This is writing $[\gamma]^2$ as a product of 3 torsion elements

What values can stl take? π? $\sqrt{2}$?

$\boxed{\text{Thrm}}$ [A., Chen]

$G = A * B$ where A and B are finite abelian groups. Then, for any $g \in G$, $\mathrm{stl}(g)$ is rational and computable.

<u>Proof</u>: Turn this into a geometry problem and then use linear programming.

$\boxed{\text{Thrm}}$ [A., Chen]

$G = \langle a, b \mid a^p = b^q = 1 \rangle = \mathbb{Z}/p * \mathbb{Z}/q$

$p \leq q$. Then:

$$\mathrm{stl}(ab) = 1 - \frac{q}{p(q-1)}$$